2019 BMEN3010 Review

VECTORS + TENSORS

KRONECKER DELTA (δ_{ij})

$$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$$

Ex. 1 (from HW): EVALUATE δ_{ii}

SOLN:
$$\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33}$$
$$= 1 + 1 + 1$$
$$= 3$$

Ex. 2: EVALUATE $\underset{\sim}{\nabla} \cdot \underset{\sim}{x}$ WHERE $\underset{\sim}{x} = x_i \underset{\sim}{e}_i$

SOLN:
$$= \frac{\partial}{\partial x_i} \underset{\sim}{e}_i \cdot x_j \underset{\sim}{e}_j$$

$$= \frac{\partial x_j}{\partial x_i} \underset{\sim}{e}_i \cdot \underset{\sim}{e}_j$$

$$= \frac{\partial x_j}{\partial x_i} \delta_{ij}$$

"contraction"

$$= \frac{\partial x_i}{\partial x_i}$$

$$= \frac{\partial x_1}{\partial x_1} + \frac{\partial x_2}{\partial x_2} + \frac{\partial x_3}{\partial x_3} = 3$$

DYADIC PRODUCT $\otimes$ <DEFINE>

Ex 3: EVALUATE $\underset{\sim}{\nabla} \otimes \underset{\sim}{x}$ where $\underset{\sim}{x} = x_i \underset{\sim}{e_i}$

$$\frac{\partial}{\partial x_i} \underset{\sim}{e_i} \otimes x_j \underset{\sim}{e_j} = \frac{\partial x_j}{\partial x_i} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j}$$

$$= \delta_{ij} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j}$$

$$= \underset{\approx}{\delta} \text{ (IDENTITY TENSOR)}$$

Ex 4: SIMPLIFY $\underset{\approx}{A} \cdot \underset{\sim}{a}$

$$= A_{ij} \underset{\sim}{e_i} \otimes \underset{\sim}{e_j} \cdot a_k \underset{\sim}{e_k}$$

$$= A_{ij} a_k \underset{\sim}{e_i} \delta_{jk}$$

$$= A_{ij} a_j \underset{\sim}{e_i} \quad \text{(a vector)}$$

$$= A_{11} a_1 \underset{\sim}{e_1} + A_{12} a_2 \underset{\sim}{e_1} + A_{13} a_3 \underset{\sim}{e_1} + \ldots$$

LEVI CIVITA SYMBOL

$$\epsilon_{ijk} = \begin{cases} 1 & \text{cyclic} \\ -1 & \text{anti cyclic} \\ 0 & \text{otherwise} \end{cases}$$

$$\underset{\sim}{a} \times \underset{\sim}{b} = a_i b_j \epsilon_{ijk} \underset{\sim}{e_k}$$

Ex 5: PROVE $\underset{\sim}{u} \times v = -v \times u$ (from HW1)

$$\underset{\sim}{u} \times \underset{\sim}{v} = u_i v_j \epsilon_{ijk} \underset{\sim}{e_k}$$
$$= -u_i v_j \epsilon_{jik} \underset{\sim}{e_k}$$
$$= -\underset{\sim}{v} \times \underset{\sim}{u}$$

<u>Ex 6</u>: Prove $\underset{\sim}{\nabla} \cdot (\underset{\sim}{\nabla} \times \underset{\sim}{v}) = 0$

$$= \frac{\partial}{\partial x_i} \underset{\sim}{e_i} \cdot \frac{\partial}{\partial x_j} \underset{\sim}{e_j} \times v_k \underset{\sim}{e_k}$$
$$= \frac{\partial^2 v_k}{\partial x_i \partial x_j} \underset{\sim}{e_i} \cdot \underset{\sim}{e_j} \times \underset{\sim}{e_k}$$
$$= \frac{\partial^2 v_k}{\partial x_i \partial x_j} \underset{\sim}{e_i} \cdot \epsilon_{jkl} \underset{\sim}{e_l}$$
$$= \frac{\partial^2 v_k}{\partial x_i \partial x_j} \epsilon_{jkl} \delta_{il}$$
$$= \frac{\partial^2 v_k}{\partial x_i \partial x_j} \epsilon_{ijk}$$

$= 0$ because $\frac{\partial^2 v_k}{\partial x_i \partial x_j} = \frac{\partial^2 v_k}{\partial x_j \partial x_i}$ so each term IN EXPANSION WILL CANCEL OUT W/ ITS ANTICYCLIC PERMUTATION.

<u>COORDINATE TRANSFORMATIONS</u> - ASK IF QUESTIONS AND REVIEW ISOLATIN COMPONENTS BY DOT PROD.

$$\underset{\sim}{v} = v_i \underset{\sim}{e_i} = v_j' \underset{\sim}{e_j'}$$

$$\underset{\sim}{v} \cdot \underset{\sim}{e}_j' = v_j' \quad \text{etc.}$$

STATICS

– 1) SET UP COORD. SYS
2) FREE BODY DIAGRAM
3) $\Sigma F = \Sigma M = 0$

EX 7: <Insert Statics problem>

STRAIN

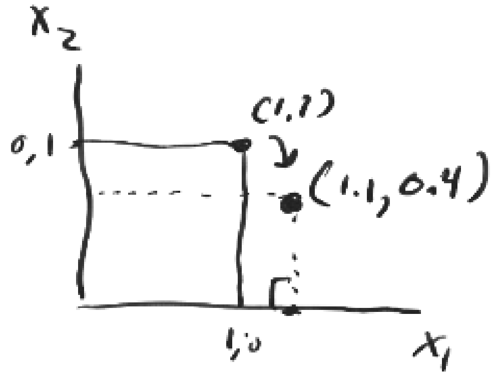
EX. 8 A UNIT CUBE IS SUBJECT TO STRAIN IN 2D

$$\underset{\approx}{\epsilon} = 0.1\, \underset{\sim}{e}_1 \otimes \underset{\sim}{e}_1 + -0.6\, \underset{\sim}{e}_2 \otimes \underset{\sim}{e}_2$$

A) SKETCH THE DEFORMED SHAPE IN 2-D

B) IF MATERIAL IS INCOMPRESSIBLE FIND $\underset{\sim}{e}_3 \cdot \underset{\approx}{\epsilon} \cdot \underset{\sim}{e}_3$, ASSUMING NO SHEAR

ANSWER: A)



B) Volume = constant = 1

$$(0.4)(1.1)(dx_3) = 1$$

$$dx_3 = 2.27$$

$$\underset{\sim}{e_3} \cdot \underset{\approx}{\epsilon} \cdot \underset{\sim}{e_3} = \epsilon_{ij}\, \underset{\sim}{e_3} \cdot \underset{\sim}{e_i} \otimes \underset{\sim}{e_j} \cdot \underset{\sim}{e_3}$$

$$= \epsilon_{ij}\, \delta_{3i}\, \delta_{3j}$$

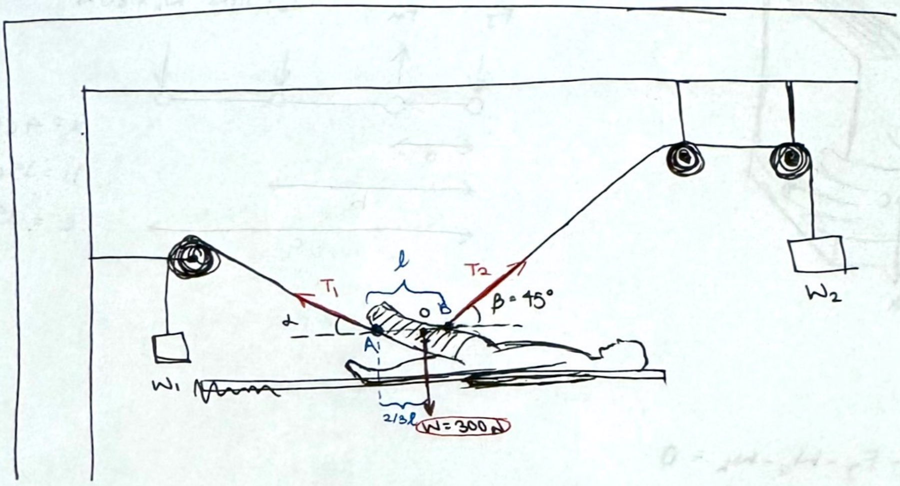
$$= \epsilon_{33}$$

$$= \frac{\Delta l}{l_0} = \frac{2.27-1}{1}$$

$$\boxed{\epsilon_{33} = 1.27}$$

Split Russel traction device: leg held by 2 weights connected to leg via 2 cables. Combined weight of leg + cast is 300 N. Horizontal distance between points A and B is l, and center of gravity is 2/3 of l along AB as measured from A. Angle β made by cable 2 is 45° to horizontal.

Find T_1, T_2, w_1, w_2, α



1) $\Sigma F_x = 0$

2) $\Sigma F_y = 0$

3) $\Sigma M_o = 0$

O is COM of leg

- can choose other moments but this one makes calcs. easier!

$\underset{\sim}{M} = \underset{\sim}{r} \times \underset{\sim}{F}$

1. $-T_{1x} + T_{2x} = 0$

$-T_1 \cos\alpha + T_2 \cos(45°) = 0$

$T_2 = \dfrac{T_1 \cos\alpha}{\cos(45°)}$

2. $T_{2y} + T_{1y} - W = 0$

$T_2 \sin(45°) + T_1 \sin\alpha - 300\text{ N} = 0$

$T_2 \sin(45°) + T_1 \sin\alpha = 300\text{ N}$

★ Only taking vertical comp. here bc $\underset{\sim}{M}$ = force * ⊥ distance from point

↳ If line of action passes through point O, then it contributes no torque — in this case true for horiz. components

$W(0l) +$

3. $T_2 \sin(45°)\left(\frac{1}{3}l\right) - T_1 \sin\alpha\left(\frac{2}{3}l\right) = 0$

$T_2 = \dfrac{2T_1 \sin\alpha}{\sin(45°)}$

Plug (3) into (1)

$\dfrac{2\cancel{T_1}\sin\alpha}{\sin(45°)} = \dfrac{\cancel{T_1}\cos\alpha}{\cos(45°)}$

$\dfrac{2\sin\alpha}{\cos\alpha} = \dfrac{\sin(45°)}{\cos(45°)}$

$2\tan\alpha = \tan(45°)$

$\alpha = \tan^{-1}\left(\frac{1}{2}\tan 45°\right)$

$\boxed{\alpha = 26.57°}$

Plug solved T_1 and α into (1)

$T_2 = \dfrac{T_1 \cos\alpha}{\cos(45°)}$

$T_2 = \dfrac{(223.6\text{N})\cos(26.57°)}{\cos(45°)}$

$\boxed{T_2 = 282.83\text{N}} = W_2$

Plug (3) into (2) + solved α

$\dfrac{2T_1\sin\alpha}{\cancel{\sin(45°)}}\left(\cancel{\sin(45°)}\right) + T_1 \sin\alpha = 300\text{N}$

$T_1(2\sin\alpha + \sin\alpha) = 300\text{ N}$

$T_1 = \dfrac{300\text{ N}}{2\sin(26.57°) + \sin(26.57°)}$

$\boxed{T_1 = 223.6\text{ N}} = W_1$